

# Large Neutrino Mixing with Universal Strength of Yukawa Couplings

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We analyse, within the framework of universal strength for Yukawa couplings (USY), various structures for the Dirac and Majorana neutrino mass matrices giving rise, through the see-saw mechanism, to a degenerate mass spectrum. A specific USY ansatz is presented for the charged lepton and neutrino effective mass matrix, leading to quasi-degenerate neutrinos and a leptonic mixing matrix which provides a large angle solution for both the atmospheric and solar neutrino problems.

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## I. Introduction

The measurement of solar and atmospheric neutrino fluxes provides experimental evidence pointing towards neutrino oscillations, thus implying non-zero neutrino masses and leptonic mixing. These exciting results have motivated various attempts at understanding the structure of neutrino masses and mixing [1]. Assuming three neutrinos, the required neutrino mass differences are such that in order for neutrinos to be of cosmological relevance, their masses have to be approximately degenerate. For Majorana neutrinos the case of quasi-degeneracy is specially interesting, since mixing and CP violation can occur even in the limit of exact mass degeneracy [2].

In this paper, we propose a simple ansatz within the framework of universal strength for Yukawa couplings (USY) [3] which leads in a natural way to a set of highly degenerate neutrinos, while providing a large mixing solution for both the solar and atmospheric neutrino data. Within USY, all Yukawa couplings have equal moduli, but different complex phases, thus leading to complex unimodular mass matrices. We extend this idea to the leptonic sector, choosing ansätze where the charged lepton and neutrino mass matrices have this special form. In the quark sector the USY hypothesis already proved to be quite successful, leading to ansätze for the Yukawa couplings, where the parameters of the Cabibbo-Kobayashi-Maskawa matrix are predicted in terms of quark mass ratios, without any free parameters [4].

The most recent results of the SuperKamiokande (SK) collaboration [5] [6] strengthen the possibility of nearly maximal mixing angle for atmospheric neutrino oscillations with the experimental parameters within the range [7]  $\Delta m_{atm}^2 = (1.5 - 8) \times 10^{-3} \text{ eV}^2$ ,  $\sin^2(2\theta_{atm}) > 0.8$ . In the absence of sterile neutrinos the dominant mode is  $\nu_\mu \longleftrightarrow \nu_\tau$  oscillations while the sub-leading mode  $\nu_\mu \longleftrightarrow \nu_e$  is severely restricted by the SK and CHOOZ [8] data which require  $V_{13} \leq 0.2$  for the range given above.

The interpretation of the present solar neutrino data leads to oscillations of the electron neutrino into some other neutrino species, with three different ranges of parameters still allowed. In the framework of the MSW mechanism [9] there are two sets of solutions, the adiabatic branch (AMSW) requiring a large mixing, ( $\Delta m_{sol}^2 = (2 - 20) \times 10^{-5} \text{ eV}^2$ ,  $\sin^2(2\theta_{sol}) = 0.65 - 0.95$ ) [6] [10], and the non-adiabatic branch (NAMSW) requiring small mixing ( $\Delta m_{sol}^2 = 5.4 \times 10^{-6} \text{ eV}^2$ ,  $\sin^2(2\theta_{sol}) \sim 6.0 \times 10^{-3}$ , for the best fit) [10]. In the framework of vacuum oscillations again large mixing is required ( $\Delta m_{sol}^2 = 8.0 \times 10^{-11} \text{ eV}^2$ ,  $\sin^2(2\theta_{sol}) = 0.75$ , for the best fit) [10].

The result of the LSND collaboration based on a reactor experiment [11] has not yet been confirmed by other experiments and in particular the KARMEN data [12] already excludes a sizeable part of the allowed parameter space. In this paper, we only take into consideration the solar and atmospheric neutrino data and consider three neutrino families without additional sterile neutrinos.

The paper is organized as follows. In the next section, we analyse various possibilities for having a degenerate or quasi-degenerate neutrino mass spectrum in USY, within the framework of the see-saw mechanism. In section III, we present a specific USY ansatz for charged lepton and neutrino mass matrices. In section IV, we show through

numerical examples, how the ansatz can accommodate all present data on atmospheric and solar neutrinos. Finally our conclusions are presented in section V.

## II. The See-saw mechanism and USY

The see-saw mechanism provides one of the most attractive scenarios for having naturally small masses for the left-handed neutrinos. Although the mechanism has been introduced within the framework of models with an extended gauge group such as  $SO(10)$  [13] or  $SU(2)_L \times SU(2)_R \times U(1)$  [14], it is clear that one may have the see-saw mechanism within the standard  $SU(3)_C \times SU(2)_L \times U(1)$  theory, through the introduction of three right-handed neutrinos, with no other modification. The full  $(6 \times 6)$  neutrino mass matrix can be written as:

$$\mathcal{M} = \begin{bmatrix} 0 & m_D \\ m_D^T & M_R \end{bmatrix} \quad (1)$$

where  $m_D$  denotes the neutrino Dirac mass matrix, while  $M_R$  stands for the right-handed Majorana mass matrix. The Dirac mass matrix is proportional to a vacuum expectation value  $v$  of the Higgs doublet responsible for the  $SU(2) \times U(1)$  breaking, while the right-handed Majorana mass term, being invariant under  $SU(2) \times U(1)$ , has a scale  $V_o$  which can be much larger than  $v$ . The masses and mixing of the left-handed neutrinos are determined by an effective mass matrix given by:

$$m_{eff} = -m_D M_R^{-1} m_D^T \quad (2)$$

In this section, we analyse the various structures for  $m_D$  and  $M_R$ , which can lead to  $m_{eff}$  corresponding to quasi-degenerate neutrinos. We are specially interested in structures based on the USY principle. We will consider various examples, without attempting at being exhaustive. For simplicity, we will consider the exact degeneracy limit. The quasi-degenerate case can be viewed as a small perturbation around that limit.

Within the USY framework, exact mass degeneracy for a  $3 \times 3$  matrix is achieved for a mass matrix proportional to  $Y$ , where,

$$Y = \frac{1}{\sqrt{3}} \begin{bmatrix} \omega & 1 & 1 \\ 1 & \omega & 1 \\ 1 & 1 & \omega \end{bmatrix} \quad (3)$$

with  $\omega = e^{2\pi i/3}$ . It can be readily verified that  $Y$  can also be written as

$$Y = e^{\frac{5\pi i}{6}} F \cdot \text{diag}(1, 1, \omega^*) \cdot F^T \quad (4)$$

where  $F$  is given by

$$F = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ \frac{-1}{\sqrt{2}} & \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{bmatrix} \quad (5)$$

In the framework of the see-saw mechanism, there are various cases which can lead to mass degeneracy.

*Case I* Both the Dirac and the right-handed Majorana mass matrices are proportional to  $Y$  so that one obtains for the full neutrino mass matrix:

$$\mathcal{M} = \begin{bmatrix} 0 & \lambda Y \\ \lambda Y & \mu Y \end{bmatrix} \quad (6)$$

where  $\lambda, \mu$  are real constants with dimension of mass, satisfying the relation  $\lambda^2/\mu \approx v^2/V_o$ . The effective  $3 \times 3$  mass matrix is then given by

$$m_{eff} = -\frac{\lambda^2}{\mu} Y Y^{-1} Y = -\frac{\lambda^2}{\mu} Y \quad (7)$$

One concludes that if  $m_D$  and  $M_R$  are proportional to  $Y$ , then  $m_{eff}$  will also have a degenerate mass spectrum.

*Case II* Both  $M_R$  and  $M_D$  have again degenerate eigenvalues, but we assume that  $M_R$  is proportional to  $Y$  in the weak-basis where  $m_D$  is already diagonal and therefore proportional to the unit matrix. The neutrino mass matrix has then the form:

$$\mathcal{M} = \begin{bmatrix} 0 & \lambda \mathbb{1} \\ \lambda \mathbb{1} & \mu Y \end{bmatrix} \quad (8)$$

which leads to

$$m_{eff} = -\frac{\lambda^2}{\mu} Y^{-1} = -\frac{\lambda^2}{\mu} Y^* \quad (9)$$

It is clear from Eq.(4) that  $m_{eff}$  has also a degenerate mass spectrum.

*Case III* Let us now consider a situation analogous to case II, but where the forms of  $M_R$  and  $m_D$  are interchanged, i.e.

$$\mathcal{M} = \begin{bmatrix} 0 & \lambda Y \\ \lambda Y & \mu \mathbb{1} \end{bmatrix} \quad (10)$$

which implies

$$m_{eff} = -\frac{\lambda^2}{\mu} Y^2 = -\frac{\lambda^2}{\mu} i Y^* \quad (11)$$

so that one obtains again  $m_{eff}$  with a degenerate mass spectrum.

*Case IV* So far, we have only considered cases where both  $m_D$  and  $M_R$  have degenerate eigenvalues. We shall now assume that  $m_D$  has an hierarchical spectrum and show that one may obtain a  $m_{eff}$  with degenerate spectrum, using a  $M_R$  which has an hierarchical spectrum also. For definiteness, let us assume that  $m_D$  is given by

$$m_D = \lambda \begin{bmatrix} e^{i\epsilon_1} & 1 & 1 \\ 1 & e^{i\epsilon_1} & 1 \\ 1 & 1 & e^{i\epsilon_2} \end{bmatrix} \quad (12)$$

where  $\epsilon_i$  are real parameters, satisfying the relations  $|\epsilon_1| \ll |\epsilon_2| \ll 1$ . The matrix  $m_D = \lambda m_{D_o}$  can be written as a sum with

$$m_{D_o} = \Delta + \epsilon_1 A + \epsilon_2 B \quad (13)$$

where

$$\Delta = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad (14)$$

and

$$A = \frac{(e^{i\epsilon_1}-1)}{\epsilon_1} \text{diag}(1, 1, 0), \quad B = \frac{(e^{i\epsilon_2}-1)}{\epsilon_2} \text{diag}(0, 0, 1) \quad (15)$$

Since  $A, B$  are of order 1, it is clear that  $m_D$  has a hierarchical spectrum. If we choose now

$$M_R = \mu m_{D_o} Y^* m_{D_o} \quad (16)$$

one obtains

$$m_{eff} = -\frac{\lambda^2}{\mu} m_{D_o} [m_{D_o} Y^* m_{D_o}]^{-1} m_{D_o} = -\frac{\lambda^2}{\mu} Y \quad (17)$$

where we have used the fact that  $(Y^*)^{-1} = Y$ . It is clear that  $m_{eff}$  has a degenerate spectrum. The interesting point is that  $M_R$  has a hierarchical spectrum, since from Eqs.(13, 16) one obtains

$$M_R = \mu [\Delta + \epsilon_1 A + \epsilon_2 B] \cdot Y^* \cdot [\Delta + \epsilon_1 A + \epsilon_2 B] = 3e^{-\frac{\pi i}{6}} \mu [\Delta + \epsilon_1 A' + \epsilon_2 B'] \quad (18)$$

where we have used the fact that for any matrix  $Z$ , one has  $\Delta Z \Delta = (\sum_{ij} Z_{ij}) \Delta$ . It is clear from Eq.(18) that  $M_R$  has indeed a hierarchical spectrum since  $A'$  and  $B'$  are at most of order one.

We have shown that starting from a hierarchical Dirac neutrino mass  $m_D$ , it is always possible to find a Majorana mass matrix  $M_R$  which leads to an exactly degenerate mass matrix of the USY type. However, it should be stressed that in order to achieve that, it is required a significant amount of fine-tuning between the Dirac and Majorana sectors, unless there is a symmetry principle constraining both sectors.

### III. A specific ansatz within the USY framework

In this section, we suggest the following specific ansatz for the charged lepton mass matrix  $M_\ell$  and the effective  $3 \times 3$  neutrino mass matrix  $M_\nu$ :

$$M_\ell = c_\ell \begin{bmatrix} e^{ia} & 1 & 1 \\ 1 & e^{ia} & 1 \\ 1 & 1 & e^{-i(a+b)} \end{bmatrix}, \quad M_\nu = c_\nu \begin{bmatrix} e^{i\alpha} & 1 & 1 \\ 1 & e^{i\beta} & 1 \\ 1 & 1 & e^{-i(\alpha+\beta)} \end{bmatrix} \quad (19)$$

Both  $M_\ell$  and  $M_\nu$  are of the USY type, symmetric and with only three real free parameters each, thus leading to full calculability of the mixing angles in terms of the mass ratios. Note that  $M_\nu$  is the relevant mass matrix for the neutrinos; it can either be an effective see-saw mass matrix, as discussed in the previous section or simply a Majorana mass matrix for left-handed neutrinos in a model with no right-handed neutrinos.

The leptonic charged weak current interactions can be written as:

$$\mathcal{L}_W = \frac{g_W}{2} (\bar{\ell}, \bar{\nu}, \bar{\nu})_L \gamma_\mu V \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L W^\mu + \text{h.c.} \quad (20)$$

where the leptonic mixing matrix  $V$  is given by:

$$V = U_\ell^\dagger \cdot U_\nu \quad (21)$$

and where

$$\ell_{L_i}^{\text{weak}} = (U_\ell)_{ij} \ell_{L_i}^{\text{phys}}, \quad \nu_{L_\alpha} = (U_\nu)_{\alpha i} \nu_{L_i} \quad (22)$$

with  $\ell_{L_i}^{\text{phys}}$  denoting the physical charged leptons and  $\nu_{L_i}$  the physical light neutrinos. The charged leptons have hierarchical masses, thus implying that the phases  $a$  and  $b$  in Eq.(19) have to be small. These phases can be expressed in terms of the charged lepton masses and to leading order one obtains:

$$|a| = 3 \frac{m_e}{m_\tau}, \quad |b| = \frac{9}{2} \frac{m_\mu}{m_\tau} \quad (23)$$

On the other hand, we want the matrix  $M_\nu$  in Eq.(19) to lead to highly degenerate neutrino masses. It can be easily checked that the matrix

$$M = c \begin{bmatrix} e^{i\alpha} & 1 & 1 \\ 1 & e^{i\alpha} & 1 \\ 1 & 1 & e^{-i2\alpha} \end{bmatrix} \quad (24)$$

has in general two degenerate eigenvalues and that in particular, for  $\alpha = 2\pi/3$  we recover the  $Y$  matrix of Eq.(3), where all three eigenvalues are exactly degenerate. This suggests that we expand  $\alpha$  and  $\beta$  in Eq.(19) around the value  $2\pi/3$ , introducing two small parameters  $\delta$  and  $\varepsilon$  defined by:

$$\alpha = \frac{2\pi}{3} - \delta - \varepsilon, \quad \beta = \frac{2\pi}{3} - \delta \quad (25)$$

In the limit  $\varepsilon = 0$ , one still has a two-fold degeneracy of eigenvalues, as in Eq.(24). The eigenvalues  $\lambda_i$  of the dimensionless hermitian matrix  $H_\nu \equiv (M_\nu M_\nu^\dagger)/(3c_\nu^2)$  are given in terms of  $\alpha$  and  $\beta$  by the expression

$$\lambda_i = 1 + 2x \cos \phi_i \quad (26)$$

with

$$\phi_1 = \theta + \frac{\alpha-\beta}{3} - \frac{2\pi}{3}, \quad \phi_2 = \theta + \frac{\alpha-\beta}{3} + \frac{2\pi}{3}, \quad \phi_3 = \theta + \frac{\alpha-\beta}{3} \quad (27)$$

and

$$\tan(\theta) = \frac{\sin \beta - \sin \alpha}{\cos \beta + \cos \alpha + 1}, \quad x = \frac{1}{3} \sqrt{3 + 2 \cos(\beta + \alpha) + 2 \cos \beta + 2 \cos \alpha} \quad (28)$$

The parameters  $\delta$  and  $\varepsilon$  can be expressed in terms of neutrino masses, and in leading order one has:

$$|\delta| = \frac{1}{\sqrt{3}} \frac{\Delta m_{32}^2}{m_3^2}, \quad |\varepsilon| = \sqrt{3} \frac{\Delta m_{21}^2}{m_3^2} \quad (29)$$

where  $\Delta m_{ij}^2 = |m_i^2 - m_j^2|$ . The matrix  $H_\ell \equiv (M_\ell M_\ell^\dagger)/(3c_\ell^2)$  is approximately diagonalized by  $U_\ell = F$  defined in Eq.(5), with additional small corrections expressible in terms of charged lepton mass ratios. The diagonalization of  $M_\nu$  requires special care since to leading order  $M_\nu$  is an exactly degenerate mass matrix. In Ref. [2] we have studied the general form of Majorana neutrino mass matrices leading to exact degeneracy and we have pointed out that if a given unitary matrix  $U_\circ$  diagonalizes the degenerate mass matrix, so does the matrix  $U_\nu = U_\circ O$ , with  $O$  an arbitrary orthogonal matrix. The diagonalizing matrix  $U_\nu = U_\circ O$  is only fixed when the mass degeneracy is lifted. For our specific case with  $M_\nu$  given by Eq.(19), we obtain in next to leading order :

$$U_\nu = \frac{e^{-\frac{\pi i}{4}}}{\sqrt{3}} \begin{bmatrix} \omega & 1 & 1 \\ 1 & \omega & 1 \\ 1 & 1 & \omega \end{bmatrix} \cdot K \quad (30)$$

where  $K = \text{diag}(-1, 1, 1)$ , so that  $U_\nu^T M_\nu U_\nu$  is diagonal real and positive for a positive  $c_\nu$ . As a result the moduli of the mixing matrix are, to a very good approximation, given by:

$$|V| \simeq \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix} \quad (31)$$

#### IV. Confronting the data

There is a stringent bound on the parameter  $c_\nu$  of the neutrino mass matrix in Eq.(19) from neutrinoless double beta decay, which can be expressed by  $|< m >| \equiv |\sum_i U_{ei}^2 m_{\nu i}| = |m_{ee}| < 0.2 \text{ eV}$  [15], with  $m_{ee}$  denoting the entry (11) of  $M_\nu$  in the weak basis where  $M_l$  is diagonal. Taking into account Eq.(19), this immediately leads to

$$m < \sqrt{3} c_\nu \approx 0.2 \text{ eV} \quad (32)$$

so that in the case of almost degenerate neutrinos coming from the ansatz of Eq.(19), we cannot have light neutrinos with masses higher than about 0.2 eV, where  $m$  is the approximate neutrino mass.

In order to compare our ansatz with the experimental results from atmospheric and solar neutrino experiments, we must bear in mind that in the context of three left-handed neutrinos the probability for a neutrino  $\nu_\alpha$  to oscillate into other neutrinos is given by

$$1 - P(\nu_\alpha \rightarrow \nu_\alpha) = 4 \sum_{i < j} |V_{\alpha i}|^2 |V_{\alpha j}|^2 \sin^2 \left[ \frac{\Delta m_{ij}^2}{4} \frac{L}{E} \right] \quad (33)$$

where  $E$  is the neutrino energy, and  $L$  denotes the distance travelled between the source and the detector. The translation of the experimental bounds, which are given in terms of only two flavour mixing, into the three flavour mixing is simple, since in this case we have  $V_{13}$  close to zero and also  $\Delta m_{32}^2 \gg \Delta m_{21}^2$ , and we may safely identify:

$$\sin^2 2\theta_{\text{atm}} = 4 (|V_{21}|^2 |V_{23}|^2 + |V_{22}|^2 |V_{23}|^2) \quad (34)$$

$$\sin^2 2\theta_{\text{sol}} = 4 |V_{11}|^2 |V_{12}|^2 \quad (35)$$

The following examples illustrate how our ansatz fits the experimental bounds for large solar and atmospheric mixing. The first example is in the context of vacuum oscillations and the second for large mixing AMSW.

*1st Example* We choose as input the masses for the charged leptons

$$m_e = 0.511 \text{ MeV}, \quad m_\mu = 105.7 \text{ MeV}, \quad m_\tau = 1777 \text{ MeV} \quad (36)$$

which correspond to phases  $|a| = 8.61 \times 10^{-4}$  and  $|b| = 0.267$  of Eq.(19). For the neutrino sector we choose

$$m_{\nu_3} = 0.2 \text{ eV}, \quad \Delta m_{32}^2 = 5.0 \times 10^{-3} \text{ eV}^2, \quad \Delta m_{21}^2 = 1.0 \times 10^{-10} \text{ eV}^2 \quad (37)$$

thus fixing the parameters  $|\delta| = 0.0772$  and  $|\varepsilon| = 4.98 \times 10^{-9}$  of Eq.(25).

Performing an exact numerical diagonalization of the mass matrices we obtain for the leptonic mixing matrix

$$|V| = \begin{bmatrix} 0.707 & 0.707 & 6.78 \times 10^{-10} \\ 0.406 & 0.406 & 0.819 \\ 0.579 & 0.579 & 0.574 \end{bmatrix} \quad (38)$$

which from Eq.(34) and Eq.(35) translates into

$$\sin^2(2\theta_{\text{atm}}) = 0.884, \quad \sin^2(2\theta_{\text{sol}}) = 1.0 \quad (39)$$

*2nd Example* In this second numerical application, we choose

$$m_{\nu_3} = 0.2 \text{ eV}, \quad \Delta m_{32}^2 = 5.0 \times 10^{-3} \text{ eV}^2, \quad \Delta m_{21}^2 = 5.0 \times 10^{-5} \text{ eV}^2 \quad (40)$$

in agreement with the large mixing AMSW solution for the solar problem. This case corresponds to  $|\delta| = 0.0764$  and  $|\varepsilon| = 2.48 \times 10^{-3}$ . The resulting leptonic mixing matrix coincides, to an excellent approximation, with that of Eq.(38), with the exception of  $|V_{13}|$  which is given by  $|V_{13}| = 3.38 \times 10^{-4}$ . Of course this is to be expected from the discussion of section III where we have pointed out that this ansatz implies in leading order a leptonic mixing matrix given by Eq.(31). The resulting values for  $\sin^2(2\theta_{\text{atm}})$  and  $\sin^2(2\theta_{\text{sol}})$  do not deviate from those of Eqs.(39).

In these examples, we fixed the parameters of our ansatz in such a way that we reproduce the charged leptonic masses and obtain almost degenerate neutrino masses obeying the current experimental bounds on neutrino mass splitting. The ansatz then leads to large values for  $\sin^2(2\theta_{\text{atm}})$  and  $\sin^2(2\theta_{\text{sol}})$ . Comparing our results with the experimental constraints, we conclude that our ansatz is in better agreement with the vacuum oscillation solution for solar neutrinos than with AMSW solution since the value  $\sin^2(2\theta_{\text{sol}}) = 1.0$  is disfavoured in the framework of AMSW [16]. However, it should be noted that it is possible, within USY, to obtain a  $\sin^2(2\theta_{\text{sol}})$  compatible with the AMSW solution, with a slight modification of our ansatz, by replacing  $M_\nu$  in Eq.(19) by  $M'_\nu = C \cdot M_\nu \cdot C$ , where  $C = \text{diag}(1, e^{i\alpha}, 1)$ . In this case, we obtain, for  $\alpha = -0.2$ ,  $\sin^2(2\theta_{\text{sol}}) = 0.974$ ,  $\sin^2(2\theta_{\text{atm}}) = 0.934$ , with the same mass splittings as before. This result is inside the 90% C.L. experimental value for the  $\sin^2(2\theta_{\text{sol}})$  in AMSW.

Concerning the stability of our ansatz under the renormalization group equations (RGE), we find that the VO solution is unstable, since the required  $\Delta m_{21}^2$  mass splitting, of the order of  $10^{-10} \text{ eV}^2$ , is much smaller than the mass splitting  $\Delta m_{RGE}^2$  generated by the running of the RGE. Indeed, in the framework of the standard model, one finds [17]:  $\Delta m_{RGE}^2 \approx m_\nu^2 \epsilon$ , with  $\epsilon$  given by,

$$\epsilon = \frac{Y_\tau^2}{32\pi^2} \log \left( \frac{\Lambda}{M_Z} \right) \quad (41)$$

where  $Y_\tau$  is the  $\tau$  Yukawa coupling (at  $M_Z$ ). It is clear, that even for  $\Lambda = o(10 \text{ TeV})$ ,  $\epsilon \geq o(10^{-6})$ . Therefore, with our  $m_\nu = 0.2 \text{ eV}$ , the mass splitting coming from the RGE is always larger than  $10^{-8} \text{ eV}^2$ , and this far exceeds the required VO mass splitting of  $10^{-10} \text{ eV}^2$ . This result is in agreement with previous analysis found in the literature [17], and it implies that in order to have a stable VO solution, one would need a mechanism, like e.g. an exact symmetry, which would protect  $\Delta m_{21}^2$  from becoming too large.

On the other hand, the MSW solution is quite stable. Even if we take  $\Lambda = o(10^{19}) \text{ GeV}$ , we get  $\epsilon \leq 10^{-5}$ , leading to a RGE mass splitting of  $\Delta m_{RGE}^2 \approx 10^{-7} \text{ eV}^2$ , which is much smaller than the required AMSW splitting of  $\Delta m_{21}^2 = 5 \times 10^{-5} \text{ eV}^2$ . The mixing angles are not significantly altered by the running of the RGE.

## V. Conclusions

Within the framework of the USY hypothesis, we have analysed various structures for the Dirac and Majorana neutrino mass matrices which can lead, through the see-saw mechanism, to an effective neutrino mass matrix for the left-handed neutrinos, with a degenerate mass spectrum. The physically relevant case of quasi-degenerate neutrinos can be viewed as a small perturbation of this limit. In one of the cases considered, the neutrino Dirac mass matrix has a hierarchical spectrum, but the resulting effective neutrino mass matrix has a degenerate spectrum. This case has the attractive feature of having all fermions, namely quarks, charged leptons and neutrinos with hierarchical Dirac masses. We have then put forward an USY ansatz for the charged lepton and neutrino effective mass matrix, which leads to three quasi-degenerate neutrinos. The ansatz is highly predictive since the leptonic mixing matrix is given

in leading order by a fixed matrix (independent of the lepton masses) with small corrections given in terms of lepton mass ratios, with no arbitrary parameters. A large mixing solution is obtained both for the solar and atmospheric neutrino data.

We have verified that the VO solution is unstable under the RGE, while the AMSW is stable. However, it should be pointed out that the problem of stability cannot be separated from the question of obtaining the USY structure from a symmetry principle. At present, this is still an open question, and therefore our ansatz should be viewed as an effective theory at low energies, resulting hopefully from an appropriate structure for the Dirac and right-handed Majorana neutrinos, imposed at a high energy scale.

One of the salient features of the ansatz of Eq.(19) is the fact that the mass matrices for both the charged leptons and the neutrinos have analogous structures, with all matrix elements with equal modulus and the non-vanishing phases appearing only along the diagonal. The drastic difference between the resulting spectra for the charged leptons and the neutrinos has to do with the fact that in the case of charged leptons the phases along the diagonal are small, while in the case of neutrinos the phases are close to  $2\pi/3$  which corresponds to the exact degeneracy limit. These simple structures for the mass matrices and Yukawa couplings do suggest the existence of a symmetry principle leading to them.

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